

Spectral Dispersion Curves of Polymeric Birefringent Textile Fibers

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ABSTRACT: Optical Fourier transform (OFT) polarizing microscopy combined with variable-wavelength interferometry (VAWI) was used for the determination of the spectral dispersion curves of birefringent textile fibers. We observed that the maximally bright circular fringe from the OFT technique corresponded to the anticoincidence case of the fiber fringe with the empty fringe from the VAWI technique and that the maximally dark fringe from the OFT technique corresponded to the coincidence case. With this combination of the two techniques, we identified the positions of the anticoincidence and coincidence when the VAWI technique was applied to overcome the difficulties of manually determining these positions. The intensity of the OFT patterns at the center was grabbed with a charge couple device camera and analyzed with a photodiode electrical circuit. The new observation was used for the determination of the spectral dispersion curves of the birefringence of polyethylene and aramide fibers. Microinterferograms are provided as illustrations. © 2002 Wiley Periodicals, Inc. *J Appl Polym Sci* 84: 2481–2488, 2002

Key words: variable-waveform interferometry; optical Fourier transforms; birefringence; spectral dispersion curves

INTRODUCTION

Interference microscopy has long been an invaluable technique for analyzing polymeric birefringent textile fibers.^{1–4} It is ideally suited for measuring local refractive indices, which are generally good indicators of molecular orientation and density fluctuations. The optical Fourier transform (OFT) technique, which was discovered by Pluta,² is one of the most successful approaches. Pluta observed that a cylindrical birefringent fiber can produce a characteristic OFT pattern in the back focal plane of the microscope objective when it is oriented diagonally between two crossed polarizers and is transilluminated by co-

herent light. This pattern, consisting of circular or annular dark interference fringes, depends on the wavelength of light. With variations in the wavelength of light in the visible region, starting from the long-wavelength region and passing toward the short-wavelength region ($\lambda_s = \lambda_2 < \lambda_1$, $\lambda_3 < \lambda_1$, ..., $\lambda_N < \lambda_1$), the flow of the interference pattern and its annular dark interference fringes of consecutive orders can be observed.

The interference pattern observed in the objective exit pupil behaves as OFTs. The mathematical expressions of OFTs for describing the microscopic images of polymeric textile fibers have been previously described.^{5,6} The OFT technique does not require the mounting of highly birefringent fibers in immersion media as usually necessary for conventional polarizing interference microscopy or microinterferometry of textile fibers.⁷ Moreover, the OFT technique not only is used to

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determine the interference order of the interference pattern but also permits us to rapidly identify the possible structural and geometrical micro-defects of the birefringent fibers.^{8,9}

The variable-wavelength interferometry (VAWI) technique is especially recommended for measuring the cylindrical objects of highly oriented polymer textile fibers surrounded by an air medium.^{10–12} One of the basic advantages of this technique is the simplicity of identifying the interference order for large optical path differences. The VAWI method may be useful for measuring the birefringence of textile fibers,¹³ especially those characterized by a high degree of extension.

The calculation of dispersion parameters with the VAWI technique depends primarily on the identification of the positions of the coincidence and anticoincidence cases¹⁰ when the light wavelength is varied from the red to blue regions of visible light.

In this study, interesting Fourier transform patterns of textile fibers that could be seen in the back focal plane of the microscope objective were investigated. We used the patterns in conjunction with the VAWI technique to overcome the difficulties of determining the positions of the coincidence and anticoincidence of the fringe shifts of the fiber from the VAWI technique.

OPTICAL SYSTEM

A standard (but slightly modified) Biolar PI polarizing interference microscope was used.⁷ The modification consisted of adding a slit subcondenser diaphragm, a wedge interferometer interference filter, and a double-refracting Wollaston prism. The prism was not necessary for the observation of the OFTs of the birefringent fibers but was used to measure the light wavelengths in real time with calibration plot $b(\lambda)$ (see Fig. 5 in ref. 14), where b is the interfringe spacing produced by the Wollaston prism. Figure 1 shows a schematic diagram of the polarizing interference microscope^{1,2} used for the observation of the OFT [Fig. 1(b)] and VAWI [Fig. 1(a)] patterns. When OFTs were observed, the Wollaston prism was removed from the path of light, and the slit diaphragm was located in the front focal plane of the substage condenser [Fig. 1(b)]. Therefore, the light incident on the birefringent fiber consisted of parallel beams. The birefringent fiber that oriented diagonally between two crossed polarizers acted as a specific bifocal cylindrical lens. A Bertrand lens was used along with the ocular for the

observation of conoscopic images of the birefringent fibers. It transferred the exit pupil of the microscope objective into the primary image plane where the field diagram of the ocular was placed. This lens was used either for visually observing the OFT pattern of the birefringent fiber or for grabbing the image with a charge couple device (CCD) camera.

When the Bertrand lens was removed from the path of light, a normal microscopic image of the fiber under study arose in the image plane of the objective. This image was observed through the eyepiece. Inserting the Wollaston prisms W_0 and W_2 in the path of the light beam, we applied the VAWI technique as shown in Figure 1(a). The CCD camera was used to capture the image in the case of OFT patterns. A photodiode electrical circuit was used to identify the maximum brightness and darkness of the OFT pattern center when monochromatic light of continuously variable wavelength (the VAWI technique) was used.

When the microscope was adjusted for the observation of images in the case of VAWI patterns at the maximally dark annular fringe in the OFT pattern, we observed that the interference fringe shifts in the image of the fiber coincided with the empty fringe field (i.e., coincidence case). At the maximally bright circular fringe, there was a case of anticoincidence of the fiber fringe with the empty fringe. Decreasing the light wavelength repeated the sequence of maximum darkness and brightness of the interference OFT patterns, which also repeated the sequences of coincidence and anticoincidence of the VAWI technique. Moreover, the number of these sequences of coincidence and anticoincidence cases within the visible spectrum depended on the fiber birefringence.

THEORETICAL CONSIDERATION OF THE COMBINATION OF THE OFT AND VAWI TECHNIQUES

A plane, linearly polarized wave of monochromatic light leaves the condenser and passes through the birefringent object. Because of the birefringence of the object, two spherical waves are produced. They are polarized at right angles whose radii of curvature are slightly different. When passing the analyzer, the wave fronts interfere with each other, producing an annular or circular interference pattern in the exit pupil of the microscope objective. The intensity distribu-

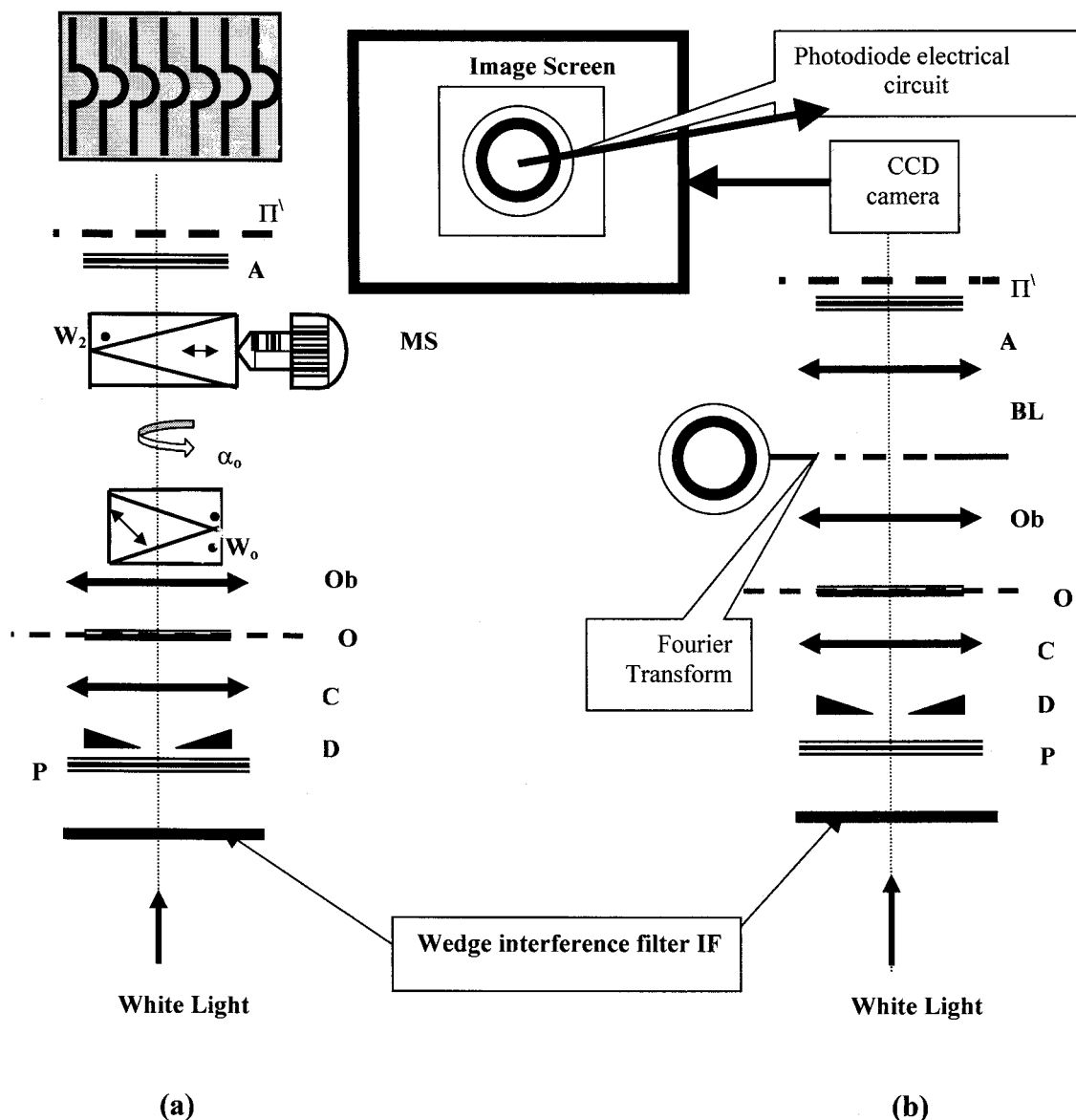


Figure 1 Schematic diagram of the optical system:^{1,2} (a) Pluta's double-refracting interference system and (b) the microscope system used for observation and processing of the OFT of the birefringent fibers: A, analyzer; BL, Bertrand lens; C, condenser; D, slit diaphragm; MS, micrometer screw; O, object; Ob, objective; P, polarizer; PhEC, photodiode electrical circuit; W₀ and W₂, birefringent prisms; Π, object plane; and Π', image plane.

tion (I) of the interference pattern may be approximately expressed as follows⁴:

$$I = I_{\max} \sin^2(\phi/2) = I_{\max} \sin^2(\pi\delta/\lambda) \quad (1)$$

where ϕ and δ are the phase shift and optical path difference, respectively, between the light components (\parallel and \perp) due to the fiber birefringence ($\delta = \phi\lambda/2\pi$).

The relationship between these interference patterns, on the one hand, and the birefringence of the fiber (B) under study and the wavelength λ of monochromatic light used, on the other hand, make it possible to determine the spectral dispersion of the birefringence $B(\lambda)$, or the discrete birefringence for given light wavelengths.

Starting from a long wavelength permits us to select a particular wavelength λ_1 for which the

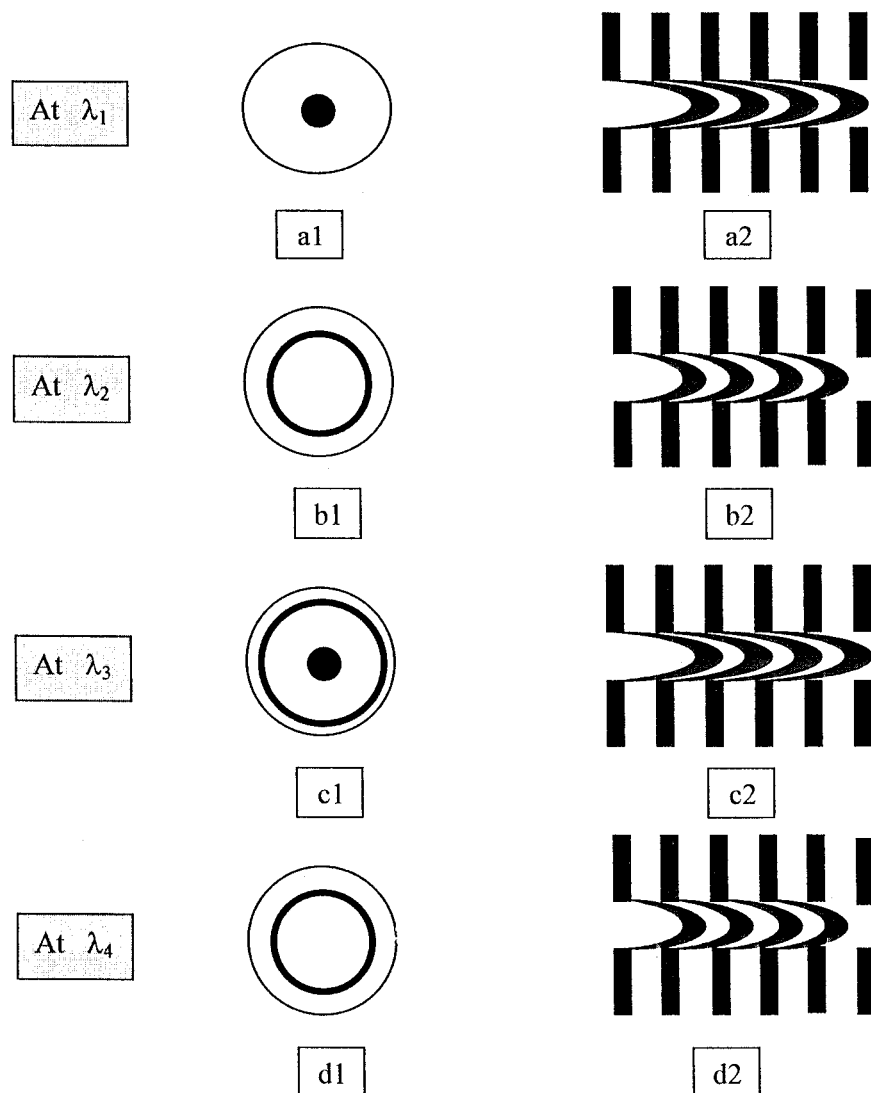


Figure 2 Illustration of the principle of measuring the spectral dispersion of polymer fibers with a combination of OFT and VAWI techniques.

center of the interference pattern is maximally dark (the coincidence case of the VAWI technique), as shown in Figure 2(a1,a2). This situation can be expressed as follows:

$$\delta_1 = 2rB_1 = m_1\lambda_1 \quad (2)$$

where m_1 is an integer called the initial interference order and r is the fiber radius. Further decreasing the wavelength by transverse sliding of the wedge interference filter leads to another particular wavelength, $\lambda_2 < \lambda_1$, for which the center of the interference pattern becomes maximally bright (the anticoincidence case of the VAWI technique), as shown in Figure 2(b1,b2). By trans-

verse sliding of the wedge interference filter, other situations at particular wavelengths can be obtained, as shown in Figure 2(c1,c2,d1,d2) and such. All these situations can be described as follows⁷:

$$\delta_s = 2rB_s = (m_1 + q_s)\lambda_s = m_s\lambda_s \quad (3)$$

where $s = 2, 3, 4, \dots$ and $q_s = 0.5, 1, 1.5, \dots$. It is self-evident that q_s expresses the increment of the current interference order: $q_1 = 0$ for $s = 1$ and $m_s = (m_1 + q_s)$.

In this work, it is necessary to select a certain particular wavelength, $\lambda_s = \lambda_2 < \lambda_1$, $\lambda_3 < \lambda_1$, ..., $\lambda_N < \lambda_1$, for which an annular dark fringe be-

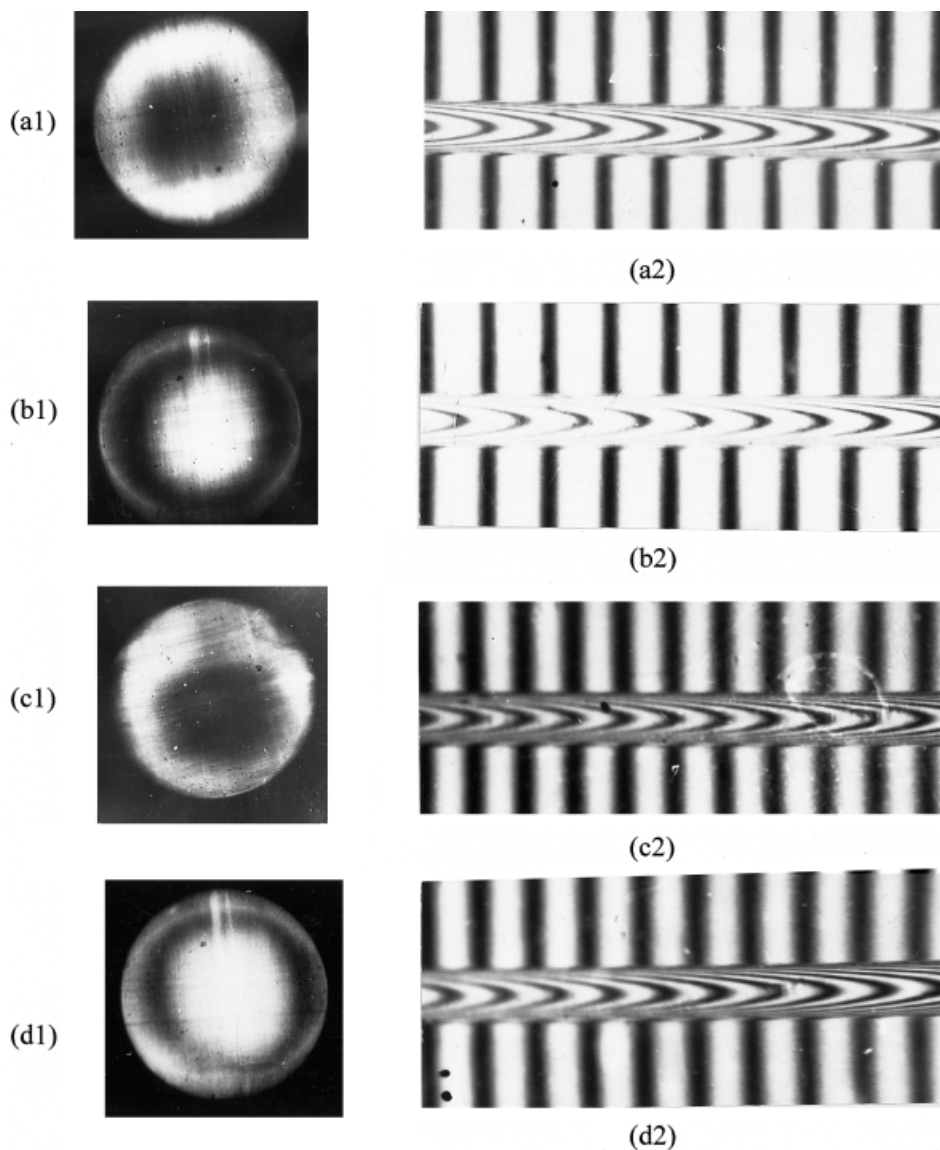


Figure 3 Sequences of microinterferograms of aramide fibers at different wavelengths with OFT and VAWI techniques.

comes consecutively maximally bright and dark in its center. For an OFT that consists of a single annular dark interference fringe, its size depends on the light wavelength λ used. The light wavelength can easily be varied with the wedge interference filter. Starting from a long-wavelength region and passing continuously toward the short-wavelength region permit us to observe a fascinating flow of the interference pattern and its annular fringes of consecutive orders. When one of the fringes appears in the marginal zone of the objective exit pupil for red light, the diameter of this fringe increases with decreasing wavelength λ , and the annular fringe eventually be-

comes a circular patch at the center of the exit pupil. Further decreasing the light wavelength repeats the aforementioned sequence of the interference patterns. The overall number of these sequences within the visible spectrum depends on the fiber birefringence.

In summary, it can be concluded that the maximally dark interference of the OFT technique corresponds to the coincidence case of the VAWI technique. Also, the maximally bright interference fringe pattern of the OFT technique corresponds to the anticoincidence case of the VAWI technique. Therefore, the OFT technique is used to identify the position of the coincidence and

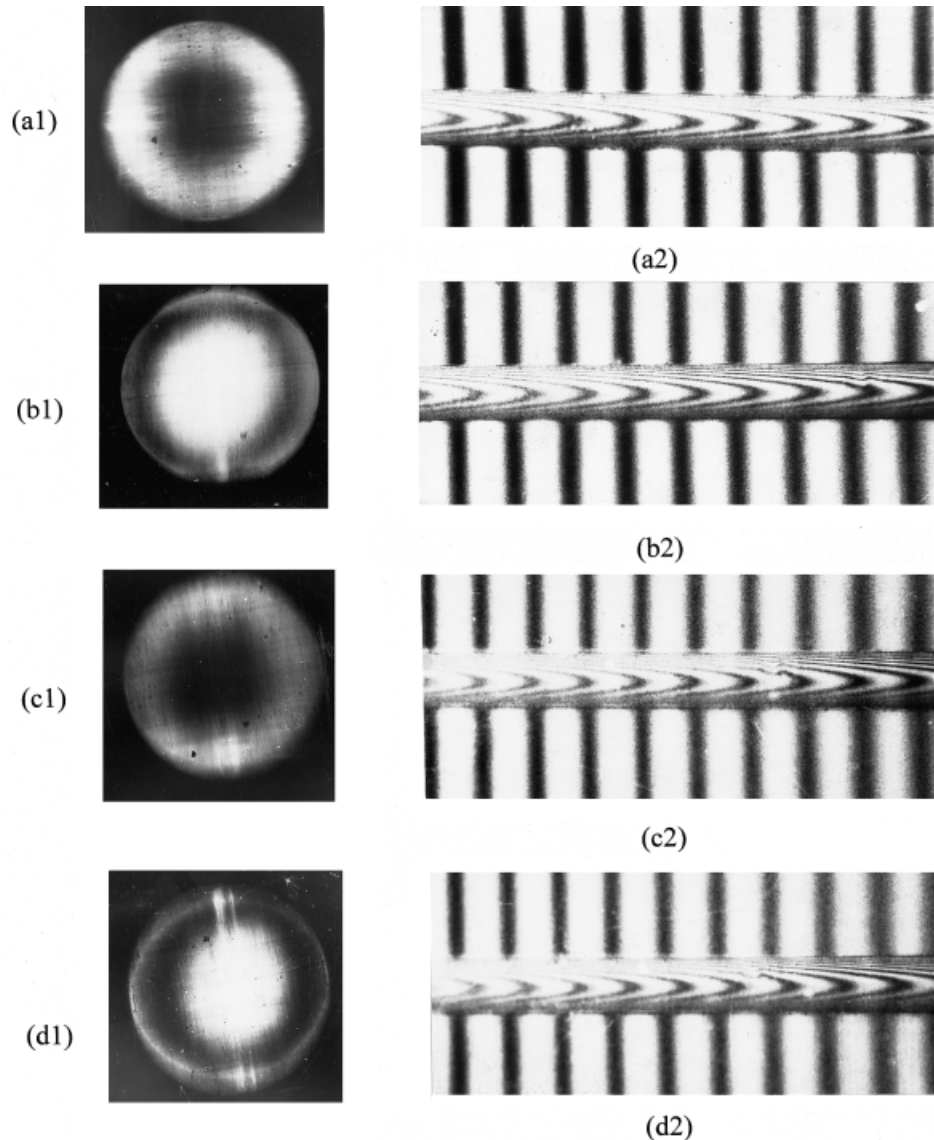


Figure 4 Sequences of microinterferograms of PE fibers at different wavelengths with OFT and VAWI techniques.

anticoincidence cases precisely when the VAWI technique is applied to determine the spectral dispersion curves of the birefringence of fibers.

Also, the equation of the initial interference order, as in the VAWI technique, is given as follows:

$$m_1 = q_s \frac{b_s}{b_1 - b_s} \quad (4)$$

where b_s is the interfringe spacing corresponding to λ_s and b_1 is the interfringe spacing corresponding to λ_1 . The subscript s denotes the coincidence number. Therefore, the OFT technique can be

used with the VAWI technique for measuring the spectral dispersion curves of the birefringence of highly oriented fibers. Figure 2 illustrates the principle of measuring the spectral dispersion of fiber birefringence with OFT and VAWI techniques.

RESULTS AND DISCUSSION

The main aim of this work is identifying the correct position of the coincidence and anticoincidence positions of the fringe in the image of the

Table I Results of the Measurement of the Birefringence of PE Fibers 23.2 μm Thick with OFT and VAWI Techniques

s	q_s	b_s (μm)	λ_s (nm)	m_1 (b)	m_1 (λ)	m_s	δ_s (μm)	B
1	0	237.50	678.372	—	—	7	4.748	0.205
2	0.5	222.58	638.981	7.46	8.11	7.5	4.792	0.207
3	1	209.75	605.088	7.56	8.26	8	4.841	0.209
4	1.5	198.43	575.191	7.62	8.36	8.5	4.889	0.211
5	2	187.90	547.385	7.58	8.36	9	4.926	0.212
6	2.5	178.24	521.885	7.52	8.34	9.5	4.958	0.214
7	3	169.00	497.473	7.40	8.25	10	4.975	0.214
8	3.5	161.21	476.902	7.39	8.28	10.5	5.007	0.216
9	4	155.00	460.501	7.52	8.45	11	5.066	0.218
10	4.5	149.93	447.101	7.70	8.70	11.5	5.142	0.222
11	5	146.00	436.732	7.98	9.04	12	5.241	0.226

fiber under study with its empty fringe field when the VAWI technique is applied visually to determine the spectral dispersion curves of the birefringence of fibers. This investigation can be performed successfully when the OFT technique is used. Then, the interference order and the spectral dispersion of the polymeric textile fiber can be determined.

The polarizing interference microscope is adjusted, as shown in Figure 1(b), for observing the OFT pattern of the birefringent fiber. The output field of the microscope is grabbed by the CCD camera and processed with a personal computer-controlled frame grabber. For the analysis of the change in the intensity at the center of the OFT pattern, the photodiode electrical circuit is used. Starting from long wavelengths permits us to select a particular wavelength λ_1 for which the center of the interference pattern is maximally dark for aramide and polyethylene (PE) fibers [Figs. 3(a1) and 4(a1), respectively]. The VAWI technique can be used for determining the light wavelength in the same time through measurement of

the interfringe spacing. Also, it is easy to correctly identify the coincidence of the fringe in the image of the fiber with its empty fringe field, as shown in Figures 3(a2) and 4(a2). Further decreasing the wavelength leads to other particular positions for which the center of the OFT pattern of these fibers becomes maximally bright. When the VAWI technique is used in the same time, the position of the anticoincidence case can be obtained.

With a calibration graph,¹⁴ the wavelengths of each sequence of the VAWI pattern were obtained. By taking the average value of $m_1(b)$, we selected the correct initial interference order. The spectral dispersion of the birefringence of the PE and aramide birefringent fibers was calculated. The results are tabulated in Tables I and II, respectively.

From Tables I and II, it is clear that the initial interference order m_1 is 7 for both PE and aramide fibers, but the dispersive power of the birefringence ($dB/d\lambda$) of aramide fiber is much greater than the dispersive power of PE fiber. Figure 5 shows the spectral dispersion curves of

Table II Results of the Measurement of the birefringence of Aramide Fibers 21.8 μm Thick with OFT and VAWI Techniques

s	q_s	b_s (μm)	λ_s (nm)	m_1 (b)	m_1 (λ)	m_s	δ_s (μm)	B
1	0	225.32	646.214	—	—	7	4.524	0.208
2	0.5	211.80	610.510	7.83	8.55	7.5	4.563	0.209
3	1	199.90	579.075	7.86	8.63	8	4.633	0.213
4	1.5	190.53	554.320	8.21	9.05	8.5	4.724	0.217
5	2	183.00	534.445	8.65	9.56	9	4.810	0.221
6	2.5	176.37	516.945	9.01	9.99	9.5	4.896	0.225
7	3	170.07	500.310	9.23	10.29	10	5.003	0.230
8	3.5	164.22	484.860	9.41	10.52	10.5	5.088	0.233
9	4	158.85	470.680	9.56	10.73	11	5.178	0.238

the birefringence of birefringent PE and aramide fibers. The slopes of these curves at any point give the dispersive power at that point.

With eq. (1), the intensity distribution of the interference pattern was calculated. Figure 6 gives the relationship between the light intensity at the center of the axially symmetrical OFT and the light wavelength for different birefringent fibers. It is clear that the number of the maximum and minimum peaks of the relationship between the light intensity and the wavelength, as shown in Figure 6, is the same as the number of the coincidence and anticoincidence positions detected for each fiber. This number depends on the birefringence of the fiber under study.

The OFT technique is combined with the VAWI technique for the determination of the spectral dispersion of the birefringence of birefringent textile fibers. In addition, this technique does not involve any immersion oils, which are usually necessary with the double-refracting Wollaston technique. Therefore, the OFT technique overcomes the problem of visually detecting the coincidence and anticoincidence of the fringe in the image of the fiber under study with its empty fringe field, especially in the short wavelength of the visible region. The only disadvantage of the OFT technique is that it cannot determine the refractive indices of fibers.

CONCLUSIONS

Spectral dispersion curves of the birefringence of polymeric PE and aramide fibers have been mea-

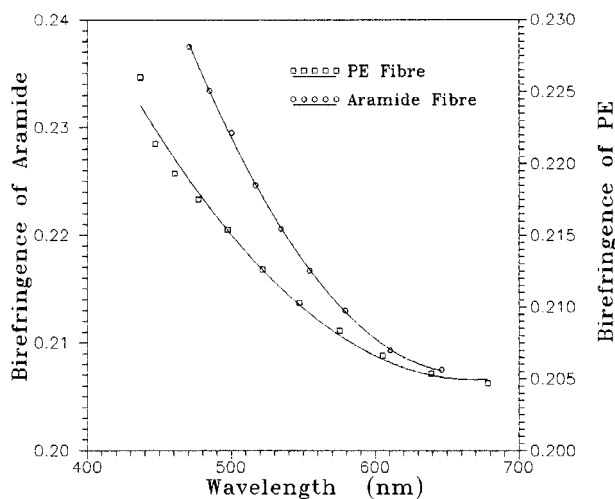


Figure 5 Spectral dispersion curves of the birefringence of aramide and PE fibers with OFT and VAWI techniques.

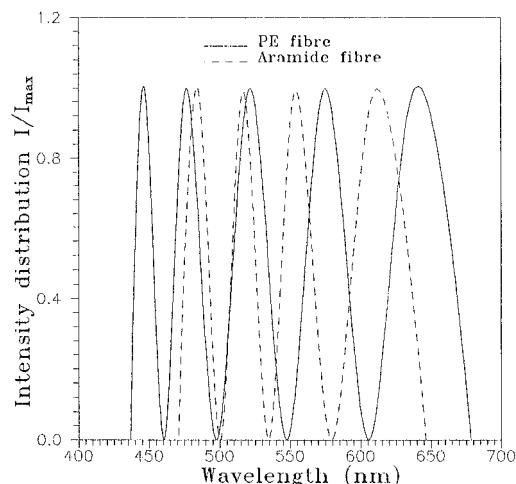


Figure 6 Relationship between the intensity and wavelength of highly birefringent fibers.

sured with a combination of the OFT and VAWI techniques. The OFT technique overcomes the problem of visually detecting the coincidence and anticoincidence positions when the VAWI technique is applied, especially in the short wavelength of the visible region. The combination of the VAWI and OFT techniques is limited only for the birefringence of fibers.

The number of the maximum or minimum peaks of the relationship between the light intensity and the wavelength, as shown in Figure 6, depends on the birefringence of the textile fibers.

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